

Transport in partially equilibrated inhomogeneous quantum wires

Alex Levchenko,¹ Tobias Micklitz,² Jérôme Rech,³ and K. A. Matveev¹

¹*Materials Science Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

²*Dahlem Center for Complex Quantum Systems and Institut für Theoretische Physik, Freie Universität Berlin, 14195 Berlin, Germany*

³*Centre de Physique Théorique, UMR 6207, Case 907, Luminy, 13288 Marseille Cedex 9, France*

(Received 30 April 2010; revised manuscript received 5 August 2010; published 8 September 2010)

We study transport properties of weakly interacting one-dimensional electron systems including on an equal footing thermal equilibration due to three-particle collisions and the effects of large-scale inhomogeneities. We show that equilibration in an inhomogeneous quantum wire is characterized by the competition of interaction processes which reduce the electrons total momentum and such which change the number of right- and left-moving electrons. We find that the combined effect of interactions and inhomogeneities can dramatically increase the resistance of the wire. In addition, we find that the interactions strongly affect the thermoelectric properties of inhomogeneous wires and calculate their thermal conductance, thermopower, and Peltier coefficient.

DOI: [10.1103/PhysRevB.82.115413](https://doi.org/10.1103/PhysRevB.82.115413)

PACS number(s): 71.10.Pm, 73.23.-b

I. INTRODUCTION

Transport properties of low-dimensional systems have been subject of intensive research work over the last two decades. One of the fundamental discoveries that has driven the field was the observation of conductance quantization in ballistic quantum wires and quantum point contacts.^{1,2} It was found that conductance exhibits a staircaselike dependence on the electron density with the universal step. The understanding of this phenomenon follows already from the single-electron picture, which predicts for the conductance of a one-dimensional single-channel clean wire,³

$$G = \frac{2e^2}{h}. \quad (1)$$

The physical origin of conductance plateaus at certain gate voltages was associated with a fixed number of occupied electronic subbands, each supplying one quantum of conductance $2e^2/h$. Within the same approach of noninteracting particles both charge and energy are carried by electronic excitations. This results in the universal relation between electric and thermal conductances, known as the Wiedemann-Franz law $K = (\pi^2/3e^2)TG$. The thermal conductance of noninteracting electrons is thus

$$K = \frac{2\pi^2}{3h}T. \quad (2)$$

In addition to G and K two thermoelectric coefficients of the electron gas are usually of great interest. These are thermopower S , which relates an induced voltage drop across the wire to applied temperature gradient, and Peltier coefficient Π connecting electric and heat currents. These two coefficients are connected by an Onsager relation $\Pi = ST$. In the absence of interactions the thermopower and Peltier coefficients are exponentially small,

$$\Pi = ST \propto e^{-\mu T} \quad (3)$$

at low temperatures $T \ll \mu$. (Here μ is the chemical potential.) The reason for such strong suppression of thermopower and Peltier coefficients is the partial cancellation between

heat currents carried by particles with energies $\mu + \epsilon$ and $\mu - \epsilon$. Only the absence of electronic states below the bottom of the band prevents Π and S from vanishing exactly.

The remarkable success of the simple single-electron picture in describing the quantization of conductance and explaining the temperature dependence of thermoelectric coefficients is attributed to the fact that quantum wires are always connected to two-dimensional leads, where interactions between electrons do not play a significant role. Even though the interactions in the wire are usually not weak, i.e., $e^2/\hbar v_F \gtrsim 1$, where v_F is the Fermi velocity, it has been shown within the so-called Luttinger-liquid model of one-dimensional electrons that the interactions inside the wire do not affect conductance quantization.⁴⁻⁶ It is no surprise then that a number of recent experiments,⁷⁻¹⁵ that revealed deviations from the perfect quantization, Eq. (1), in low-density wires, attracted a great deal of theoretical attention.¹⁶⁻²² These deviations often take the form of a shoulderlike feature, which develops at finite temperature just below the first quantized plateau, around $0.7 \times 2e^2/h$. At present there is no consensus on the theoretical interpretation of this phenomenon. However, it is generally accepted that electron-electron interaction effects should be involved in explaining these experimental observations.

In a number of recent publications^{18-20,23-37} transport properties of one-dimensional conductors were reconsidered focusing on the physics which lies beyond an ideal Luttinger-liquid model. In particular, when studying the temperature dependence of the corresponding kinetic coefficients Refs. 31-39 emphasized one fundamental aspect of interactions, namely, the role of physical processes that lead to equilibration of electrons inside the wire. It should be emphasized that equilibration is absent in an ideal Luttinger liquid since bosonic elementary excitations of the latter have infinite lifetime, thus there is no relaxation toward equilibrium in these systems, no matter how strong the interactions are. In higher-dimensional systems equilibration at low temperatures is primarily provided by pair collisions of electrons. These, however, do not provide relaxation in one-dimensional systems. This is due to the conservation laws for momentum and energy which severely restrict the phase

space available for scattering. As a result, pair collisions in ideal one-dimensional wires can occur with a zero momentum change or an interchange of the two momenta, leaving the distribution function unaffected. The leading equilibration mechanism thus involves collisions of more than two particles. For a weakly interacting system, it is then natural to assume that equilibration is provided by three-particle scattering processes.^{29–31} This, of course, also relies on the additional assumption that other degrees of freedom, which can absorb energy and momentum from electrons (phonons, for example) can be ignored. This assumption is acceptable in many cases since electron-phonon coupling constant is typically much smaller than that due to the electron-electron interactions.

In practice, long one-dimensional structures are strongly prone to inhomogeneities inevitably present due to the nearby gates or charged dopants underlying the wire. However, most preceding works studied effect of equilibration on transport assuming uniform (clean) wires. The notable exceptions include Refs. 32 and 33 where smooth inhomogeneities were accounted for while assuming full equilibration of the electronic system. The purpose of the present work is to study effects of inhomogeneities on transport properties of partially equilibrated quantum wires. We focus our attention to the situation where the scale of inhomogeneities b is much larger than the electron Fermi wavelength, $b \gg \lambda_F$, such that backscattering of electrons from the inhomogeneities is negligible. In this case only interactions (three-particle collisions) may interrupt direct flow of the electron liquid and convert (backscatter) some right-moving electrons into left-moving ones.

Since nonuniform systems are no longer translationally invariant, there is an additional scattering mechanism, which can relax electron momentum without changing the number of right- and left-moving particles.^{32,33} We find that equilibration due to three-particle collisions and inhomogeneity-induced momentum-nonconserving scattering compete with each other. An interplay between these two effects leads to a very interesting picture of the electronic transport and results in temperature-dependent corrections to the wire resistance and thermoelectric coefficients.

The paper is organized as follows. In Sec. II we discuss the general structure of the electron distribution function and transport in a clean one-dimensional wire. We briefly mention the resulting transport coefficients for this case, which were recently reported in Ref. 35. In Sec. III we develop a general formalism which enables us to treat equilibration of the electron system due to three-particle collisions and inhomogeneity-induced scattering on equal footing. The central part of our work is Sec. IV where we apply this theory to the calculation of transport coefficients in one-dimensional inhomogeneous wires. From our general expressions we recover the results known for the short and long uniform wires, and also consider several experimentally relevant simple models of inhomogeneous case. We summarize our results in Sec. V. Supplementary appendices accompany some technical aspects of our calculations.

II. TRANSPORT IN UNIFORM QUANTUM WIRES

In the absence of interactions, left- and right-moving electrons inside the wire are at equilibrium with the reservoir

they originated from. If a voltage bias V and/or temperature difference ΔT is applied between the reservoirs, then corresponding equilibria differ from each other, giving rise to a particular form of the nonequilibrium distribution function inside the wire. This distribution depends on the direction of motion of electrons and for the right- and left-moving particles is controlled, respectively, by the left and right lead,

$$f_p = \frac{\theta(p)}{e^{(\epsilon_p - \mu_l)/T_l} + 1} + \frac{\theta(-p)}{e^{(\epsilon_p - \mu_r)/T_r} + 1}. \quad (4)$$

Here $\epsilon_p = p^2/2m$ is the energy of an electron with momentum p and $\theta(p)$ is the unit step function. The difference between the chemical potentials (temperatures) in the leads is equal to the voltage (temperature difference) applied to the wire $\mu_l - \mu_r = eV$ ($T_l - T_r = \Delta T$). Using the distribution function (4) at $\Delta T = 0$ one can find electric current $I = GV$ with the conductance of noninteracting electrons $G = (2e^2/h)(1 + e^{-\mu/T})^{-1}$, thus recovering, Eq. (1), up to an exponentially small correction. The same distribution, Eq. (4), provides thermal conductance, Eq. (2), and thermoelectric coefficients $\Pi = TS = (\mu/e)e^{-\mu/T}$, consistent with Eq. (3).

In the presence of interactions ballistic propagation of electrons through the wire may be interrupted by collisions with other electrons. As a result of these collisions, some electrons change their direction of motion thus losing memory of the lead they originated from. Such backscattering processes modify the electron distribution function which is then no longer given by Eq. (4). It is important to realize that the effect of electron collisions on the distribution function depends strongly on the length of the wire. Indeed, electrons traverse short wires relatively fast, such that interactions do not have time to change distribution, Eq. (4), considerably. On the other hand, in the limit of very long wire one should expect full equilibration of left- and right-moving electrons into a single distribution, even in the case of weak interactions.

For the Galilean invariant system one can easily infer the electron distribution function in a fully equilibrated state. Indeed, viewed from a reference frame moving with the drift velocity $v_d = I/ne$ (where I is the electric current and n is the electron density) the electron system is at rest and must be described by the equilibrium Fermi distribution. Performing a Galilean transformation back into the stationary frame of reference this distribution takes the form,

$$f_p = \frac{1}{e^{(\epsilon_p - v_d p - \mu_{\text{eq}})/T_{\text{eq}}} + 1}, \quad (5)$$

where the chemical potential μ_{eq} and temperature T_{eq} inside the equilibrated wire are, in general, different from $\mu_{l(r)}$ and $T_{l(r)}$.

At zero temperature, $T = T_{\text{eq}} = 0$, the distributions, Eqs. (4) and (5), coincide, provided $\mu_{l(r)} = \mu_{\text{eq}} \pm v_d p_F$, where $p_F = \pi \hbar n / 2$ is the Fermi momentum of the system. At nonzero temperature the distribution function (5) of electrons inside the equilibrated wire is slightly different from the distribution, Eq. (4), supplied by the leads. The mismatch between the two distribution functions results in additional resistance, reducing the conductance of noninteracting electrons to³⁴

$$G_{\text{eq}} = \frac{2e^2}{h} \left[1 - \frac{\pi^2 T^2}{12 \mu^2} \right]. \quad (6)$$

This result is universal since it was obtained without making any specific assumptions regarding the process of equilibration. The corresponding derivation relied uniquely on the analysis of conservation laws for energy, momentum, and particle number. It is applicable as long as wire length L exceeds certain equilibration length ℓ_{eq} such that the distribution, Eq. (5), is already established. The exact definition of ℓ_{eq} is model specific and depends on the interaction between electrons. Quite generically, however, it can be argued that this length is exponentially large at low temperature $\ell_{\text{eq}} \propto e^{\mu/T}$. The exponential scale can be understood from the mechanism of equilibration³⁵ which is also discussed later in the text.

The thermal conductance of fully equilibrated wire is zero,

$$K_{\text{eq}} = 0. \quad (7)$$

This result can be understood simply from the structure of the distribution function (5). One should recall that thermal conductance is defined under the condition that electric current $I = env_d$ vanishes. Thus $v_d = 0$ and the distribution, Eq. (5), takes the form of the standard Fermi-Dirac distribution. Due to its symmetry $p \rightarrow -p$ the heat current carried by electrons vanishes, regardless of the temperature bias ΔT applied to the wire. One therefore finds that in an infinitely long wire the thermal conductance is zero. One should note, however, that thermal conductivity is finite,³⁵ i.e., at $L \rightarrow \infty$ the thermal conductance scales as $K \propto 1/L$.

Unlike the electric and thermal conductances, thermopower and Peltier coefficients are significantly enhanced by equilibration effects. Specifically, Π grows from the exponentially small value, Eq. (3), for short wires $L \ll \ell_{\text{eq}}$ to

$$\Pi_{\text{eq}} = TS_{\text{eq}} = \frac{\pi^2 T^2}{6e \mu}, \quad (8)$$

in fully equilibrated (long) wires, $L \gg \ell_{\text{eq}}$.

A more careful treatment of the equilibration effects is required in wires of intermediate length $L \sim \ell_{\text{eq}}$, where the electron distribution function is only partially equilibrated.³⁵ As we already mentioned, in the case of weakly interacting electrons the leading mechanism of equilibration is provided by three-particle collisions. At low temperatures one should consider two types of such collisions. The strongest scattering events involve three particles near the Fermi level (for example, one left mover that scatters off two right movers such that all particles preserve their direction of motion). These collisions are relatively fast and the corresponding scattering length ℓ_t scales as a power law of temperature. However, these collisions alone cannot establish the distribution, Eq. (5), since they conserve the number of right- and left-moving particles. The other important three-particle collisions involve backscattering of, say, a right-moving electron into a left-moving one. This backscattering occurs near the bottom of the band and provides equilibration between the chemical potentials of right and left movers, thus establishing the distribution, Eq. (5). Since at low temperatures

the probability to find an empty state at the band bottom is exponentially small, the corresponding relaxation process is very slow, and equilibration length is large, $\ell_{\text{eq}} \propto e^{\mu/T}$.

Let us consider now a segment of the wire, whose length ΔL is small compared to the equilibration length ℓ_{eq} but large as compared to ℓ_t , namely, $\ell_t \ll \Delta L \ll \ell_{\text{eq}}$. This condition implies that typical electron with energy near the Fermi level passes through the segment without backscattering so that the distribution, Eq. (5), cannot be established. On the other hand, ΔL is already sufficiently large for electrons to experience other multiple collisions which allow momentum and energy exchange between right- and left-moving electrons. Under these conditions, the electron distribution function in the segment achieves a state of partial equilibration, in which the numbers N^L and N^R of the right- and left-moving electrons are conserved independently. The form of this distribution can be obtained from the general statistical mechanics argument by maximizing the entropy of electrons while preserving $N^{L(R)}$, total energy and momentum of the system,³⁵

$$f_p = \frac{\theta(p)}{e^{(\epsilon_p - up - \mu^R)/\bar{T}} + 1} + \frac{\theta(-p)}{e^{(\epsilon_p - up - \mu^L)/\bar{T}} + 1}. \quad (9)$$

Here \bar{T} is the effective temperature, parameter u has dimension of velocity and accounts for the conservation of momentum in electron collisions, and $\mu^{L(R)}$ are the chemical potentials of the left- and right-moving particles. The distribution, Eq. (9), smoothly interpolates between the regimes of no equilibration, Eq. (4), and that of full equilibration, Eq. (5). In the absence of temperature difference, $\Delta T = 0$, the unperturbed distribution, Eq. (4), is obtained from Eq. (9) by setting $u = 0$ and identifying the chemical potentials with those in the leads: $\mu^R = \mu_l$ and $\mu^L = \mu_r$. [Here and throughout the paper we use $l(r)$ to denote left (right) lead while $L(R)$ denote left (right) movers.] The fully equilibrated distribution, Eq. (5), is obtained from Eq. (9) by setting $\Delta\mu = \mu^R - \mu^L = 0$. In this case the electric current is expressed as $I = enu$, which identifies parameter u as the drift velocity v_d . We should emphasize here that since the distribution, Eq. (9), is applicable for the segment of the wire outlined above then all four parameters $\bar{T}(x)$, $u(x)$, and $\mu^{L/R}(x)$ defining f_p are, in principle, coordinate dependent.

The implications of the distribution, Eq. (9), for the transport coefficients of partially equilibrated clean wires were discussed in Ref. 35. In the following we generalize the above picture of electronic transport in one-dimensional wires accounting for possible nonuniformities of the system.

III. TRANSPORT IN INHOMOGENEOUS QUANTUM WIRES

A. Boltzmann equation

Consider an inhomogeneous quantum wire of length L , connected by ideal reflectionless contacts to noninteracting leads and biased by a small voltage V and/or temperature difference ΔT , see Fig. 1. If the spatial variations related to inhomogeneities occur on a length scale b much larger than the Fermi wavelength λ_F , electrons do not suffer any back-

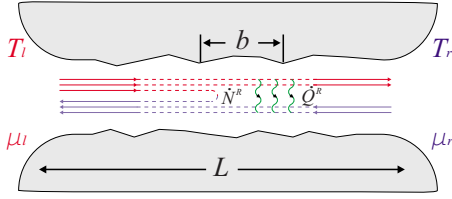


FIG. 1. (Color online) Inhomogeneous quantum wire connected adiabatically to two-dimensional leads and biased by the small voltage $\mu_l - \mu_r = eV$ and/or temperature difference $T_l - T_r = \Delta T$. We assume that the spatial scale b of the inhomogeneities is large compared to the Fermi wavelength, and thus electrons do not experience backscattering from inhomogeneities. In this case only three-particle equilibration processes may interrupt the flow of right-moving electrons and convert some right movers into left movers. Wavy lines represent heat exchange \dot{Q}^R between right movers and left movers.

scattering. Since the physical picture of equilibration in one-dimensional wire can be readily understood at the level of weakly interacting electrons we restrict our attention to this case and describe the system in the framework of kinetic equation. In this case electron distribution function $f(t, x, p)$ obeys the Boltzmann equation,

$$\partial_t f + v_p \partial_x f - \partial_x U(x) \partial_p f = \mathcal{I}\{f\}, \quad (10)$$

where static potential $U(x)$ accounts for inhomogeneities of the wire and $\mathcal{I}\{f\}$ conventionally stands for the collision integral. We are interested in the steady-state regime when distribution function $f(t, x, p)$ does not depend explicitly on time, and thus set $\partial_t f = 0$. It will be also convenient to split the distribution $f(x, p)$ into two parts corresponding to the right and left movers,

$$f(x, p) = \theta(p) f^R[x, \epsilon_p(x)] + \theta(-p) f^L[x, \epsilon_p(x)], \quad (11)$$

and express it as the function of energy $\epsilon_p(x) = p^2/2m + U(x)$ for the given momentum p . Kinetic Eq. (10) should be supplemented by the boundary conditions at the ends of the wire that are controlled by the leads,

$$f^R[l, \epsilon_p(l)] = \frac{1}{e^{[\epsilon_p(l) - \mu_l]/T_l} + 1}, \quad (12a)$$

$$f^L[r, \epsilon_p(r)] = \frac{1}{e^{[\epsilon_p(r) - \mu_r]/T_r} + 1}, \quad (12b)$$

where $\mu_l = \mu + eV$, $\mu_r = \mu$ and $T_l = T + \Delta T$, $T_r = T$. (We use shorthand notation for the distribution function of right movers at the left lead $f^R[l, \epsilon_p(l)] = f^R[x=0, \epsilon_p(x=0)]$ and left movers at the right lead $f^L[r, \epsilon_p(r)] = f^L[x=L, \epsilon_p(x=L)]$.) The parametrization, Eq. (11), is especially useful since owing to the simple algebraic relation,

$$v_p \frac{\partial f^{R(L)}}{\partial \epsilon} \frac{\partial \epsilon}{\partial x} - \frac{\partial U}{\partial x} \frac{\partial f^{R(L)}}{\partial \epsilon} \frac{\partial \epsilon}{\partial p} = 0 \quad (13)$$

the inhomogeneity-related term drops out from the left-hand side of the kinetic equation, except for the residual contribution $\delta(p) \partial_x U(x) \{f^R[x, U(x)] - f^L[x, U(x)]\}$ at $p=0$. For noninteracting electrons the mismatch between distribution func-

tions $f^{R/L}[x, U(x)]$ of right and left movers is exponentially small at the bottom of the band. In addition, even this small discontinuity is smeared by interelectron scattering responsible for equilibration.³⁵ It is thus safe to take $f^R[x, \epsilon_p(x)] = f^L[x, \epsilon_p(x)]$ for $p=0$ and we get then instead of Eq. (10),

$$\theta(p) v_p \partial_x f^R[x, \epsilon_p(x)] + \theta(-p) v_p \partial_x f^L[x, \epsilon_p(x)] = \mathcal{I}\{f\}. \quad (14)$$

As the first step of our general analysis, we demonstrate now with the help of kinetic Eq. (14) that deviations in electric and thermal conductances from their noninteracting values [Eqs. (1) and (2)] are ultimately related to the rate of change in the number of say right-moving electrons \dot{N}^R and heat exchange rate \dot{Q}^R between right movers and left movers.

B. Conservation laws

The rate of change in the number of right movers \dot{N}^R due to electron collisions is obtained from the collision integral $\mathcal{I}\{f\}$ upon integration over positive momenta and wire length,

$$\dot{N}^R = \frac{2}{h} \int_0^L dx \int_0^\infty dp \mathcal{I}\{f\}, \quad (15)$$

where the coefficient 2 stands for two spin projections. Owing to the Boltzmann equation (14) \dot{N}^R can be equivalently presented in terms of the distribution function of right-moving electrons as

$$\dot{N}^R = \frac{2}{h} \int_0^L dx \int_{U(x)}^\infty d\epsilon \partial_x f^R(x, \epsilon). \quad (16)$$

We can integrate this expression by parts by noticing that

$$\frac{\partial}{\partial x} \int_{U(x)}^\infty d\epsilon f^R(x, \epsilon) = \int_{U(x)}^\infty d\epsilon \partial_x f^R(x, \epsilon) - f^R(x, U) \partial_x U(x), \quad (17)$$

and approximating in the following the distribution function of right movers by unity at the bottom of the band, which is correct up to exponentially small terms at low temperatures $f^R(x, U) \approx 1 - \mathcal{O}[e^{-(\mu-U)/T}]$. This would give then

$$\dot{N}^R = j^R(r) - j^R(l) + \frac{2}{h} [U(r) - U(l)], \quad (18)$$

where we used standard definition for the currents of right/left movers,

$$j^{R/L}(x) = \frac{2}{h} \int_{-\infty}^{+\infty} dp \theta(\pm p) v_p f(x, \epsilon_p). \quad (19)$$

In Eq. (18) the incoming current $j^R(l)$ of right movers at $x=0$ is known since it is controlled by the distribution of noninteracting electrons in the left lead, Eq. (11). However, the outgoing $j^R(r)$ is not known because the distribution function of right movers varies along the wire as a result of scattering. It is convenient to exclude this unknown from Eq. (18) by

noticing that the total current j in the wire does not depend on position due to the conservation of the number of electrons and can be written as $j = j^R(r) + j^L(r)$. This results in

$$\dot{N}^R = j - [j^L(r) + j^R(l)] + \frac{2}{h}[U(r) - U(l)]. \quad (20)$$

The benefit of performing this step is that now both currents $j^R(l)$ and $j^L(r)$ are controlled by the noninteracting leads whose distribution functions are given by the boundary conditions [Eq. (11)]. Furthermore, since \dot{N}^R and j vanish in the absence of applied bias, we can exclude the U -dependent contribution from Eq. (20) by subtracting from it $j^L(r) + j^R(l)|_{V=0}^{\Delta T=0} = \frac{2}{h}[U(r) - U(l)]$. This leads to

$$\dot{N}^R = j - [j^R(l) - j^R(l)|_{V=0}^{\Delta T=0}]. \quad (21)$$

The difference between currents of right movers at the left boundary with and without bias in Eq. (21) can be found with the help of distribution function (11). Indeed, after a simple calculation

$$\begin{aligned} j^R(l) - j^R(l)|_{V=0}^{\Delta T=0} &= \frac{2}{h} \int_0^\infty dp v_p \{f^R[l, \epsilon_p(l)] - f^R[l, \epsilon_p(l)]|_{V=0}^{\Delta T=0}\} \\ &= \frac{2eV}{h}, \end{aligned}$$

valid up to corrections small as $e^{-\mu/T}$, we find

$$\frac{2e^2}{h} V = I - e\dot{N}^R, \quad (22)$$

where $I = ej$. This result can be thought of as a generalization of Landauer formula for interacting one-dimensional systems. In the noninteracting limit $\dot{N}^R = 0$ and we recover $G = I/V = 2e^2/h$ while a finite \dot{N}^R would lead to a change in the conductance. Equation (22) was derived earlier for uniform (clean) wires.^{34,35} We have shown here that it remains intact even in the case of inhomogeneous wires.

We now repeat the above calculation for the rate of the change of energy of the right movers \dot{E}^R induced by electron collisions. The latter is obtained from the collision integral $\mathcal{I}\{f\}$ by multiplying it by ϵ_p and then integrating over positive momenta and the wire length,

$$\dot{E}^R = \frac{2}{h} \int_0^L dx \int_0^\infty dp \epsilon_p \mathcal{I}\{f\}. \quad (23)$$

It can be equivalently rewritten in terms of $f^R(x, \epsilon)$ by making use of the Boltzmann equation (14),

$$\dot{E}^R = \frac{2}{h} \int_0^L dx \int_{U(x)}^\infty d\epsilon \epsilon \partial_x f^R(x, \epsilon). \quad (24)$$

After integration by parts, similar to Eq. (17), one finds

$$\dot{E}^R = j_E^R(r) - j_E^R(l) + \frac{1}{h}[U^2(r) - U^2(l)], \quad (25)$$

where we used the usual definition for energy currents of right/left movers,

$$j_E^{R/L}(x) = \frac{2}{h} \int_{-\infty}^{+\infty} dp \theta(\pm p) v_p \epsilon_p f(x, \epsilon_p). \quad (26)$$

Conservation of energy ensures that the energy current j_E is constant along the wire. By using it at the right end $j_E = j_E^R(r) + j_E^L(r)$, we can exclude unknown $j_E^R(r)$ from Eq. (25) in analogy with Eq. (20). In addition, since $\dot{E}^R = 0$ and $j_E = 0$ without the bias, we subtract $j_E^L(r) + j_E^R(l)|_{V=0}^{\Delta T=0} = \frac{1}{h}[U^2(r) - U^2(l)]$ to exclude the U -dependent contribution. This procedure gives for the rate of energy change,

$$\dot{E}^R = j_E - [j_E^R(l) - j_E^R(l)|_{V=0}^{\Delta T=0}]. \quad (27)$$

The energy current of right movers at the left end of the wire is controlled by the noninteracting lead with known distribution function (11). To linear order in V and ΔT a simple calculation gives us

$$\begin{aligned} j_E^R(l) - j_E^R(l)|_{V=0}^{\Delta T=0} &= \frac{2}{h} \int_0^\infty dp v_p \epsilon_p \{f^R[l, \epsilon_p(l)] - f^R[l, \epsilon_p(l)]|_{V=0}^{\Delta T=0}\} \\ &= \frac{2eV\mu}{h} + \frac{2\pi^2 T \Delta T}{3h}. \end{aligned} \quad (28)$$

It is convenient to combine electric and energy currents into the heat current,

$$j_Q = j_E - \mu j, \quad (29)$$

for which we find from Eqs. (22) and (27),

$$\frac{2\pi^2}{3h} T \Delta T = j_Q - \dot{Q}^R, \quad (30)$$

where $\dot{Q}^R = \dot{E}^R - \mu \dot{N}^R$ is the heat transferred into the right-moving subsystem by electron collisions. As expected when $\dot{Q}^R = 0$ we recover from Eq. (30) the thermal conductance of noninteracting electrons Eq. (2) while a nonvanishing \dot{Q}^R results in an interaction-induced change in K . Equation (30) was reported earlier for uniform wires^{34,35} and as it is shown here remains valid even in the inhomogeneous case.

So far our basic equations (22) and (30) contain four unknown entries: two response currents I and j_Q due to applied bias V and temperature difference ΔT , and two rates \dot{N}^R and \dot{Q}^R nonvanishing due to interactions. In the next sections we demonstrate that all four quantities can be expressed in terms of only two parameters $u(x)$ and $\Delta\mu(x) = \mu^R(x) - \mu^L(x)$ which enter the distribution function (9) of partially equilibrated electrons. This would give us the closed set of equations that relate $\{I, j_Q\} \Leftrightarrow \{V, \Delta T\}$ and thus determine the transport coefficients of interest.

C. Currents I and j_Q in the partially equilibrated wires

Electric and heat currents can be easily found knowing the distribution function $f(x, p)$ of electrons in the wire. As we discussed in Sec. II for the partially equilibrated wire this distribution is given by Eq. (9). It is worth emphasizing that for the inhomogeneous case all four parameters entering Eq. (9): velocity u , chemical potential of right- and left-moving

electrons $\mu^{R(L)}$, and effective temperature \bar{T} , are, in principle, coordinate dependent. Furthermore, the distribution, Eq. (9), does not apply to particles near the bottom of the band, for $|p| \lesssim \sqrt{mT}$, as explained later in the text (see also corresponding discussions in Ref. 35). This, however, does not cause any extra difficulties since transport quantities of interest are determined by the behavior of the distribution function near the Fermi level.

By using the definition of the current, Eq. (19), and distribution function (9) we obtain the electric current in the partially equilibrated wire

$$I = \frac{2e}{h} \Delta\mu(x) + en(x)u(x). \quad (31)$$

Notice here that although contributions to the current due to the electron drift, $en(x)u(x)$, and partial equilibration between right and left movers, $2e\Delta\mu(x)/h$, are individually coordinate dependent, their sum must be constant along the wire. This is a consequence of the particle number conservation.

Since the heat current j_Q also does not depend on position, it can be calculated at any point in the wire. In the regions not too close to the leads the distribution function is expected to have the partially equilibrated form (9). Then using expressions (19) and (26) for j and j_E we obtain after Sommerfeld expansion of the integrand to leading order in $T/\mu \ll 1$,

$$j_Q = \frac{\pi^2}{6} \frac{T^2}{\mu(x)} n(x)u(x), \quad (32)$$

where we introduced $\mu(x) = \mu - U(x)$. Since j_Q is already proportional to small $u \propto V$, we were able to replace \bar{T} with T within the linear-response regime. Notice also that the particular combination $n(x)u(x)/\mu(x)$ defining heat current j_Q must be coordinate independent.

To make further progress we should elaborate on the expressions for the rates \dot{N}^R and \dot{Q}^R , whose explicit forms depend on details of the equilibration mechanism. This can be done following the idea suggested in Ref. 35 and we show below how \dot{N}^R and \dot{Q}^R can be expressed through $\Delta\mu(x)$ and $u(x)$.

D. Microscopic expressions for \dot{N}^R and \dot{Q}^R

First let us identify the leading backscattering mechanism that contributes to \dot{N}^R . The most favorable collisions should involve a maximal number of electronic states close to the Fermi points. However, due to the conservation of total energy and momentum, collisions that change the number of right and left movers cannot occur near the Fermi level only, and have to involve states deep in the electron band. As was pointed out in Ref. 31 the scattering process most important in altering the current thus typically scatters two electrons close to the Fermi points and one electron at the bottom of the band, as schematically depicted in Fig. 2(a). It is convenient to think of this collision as a process in which a deep hole, corresponding to the outgoing electron state, is back-

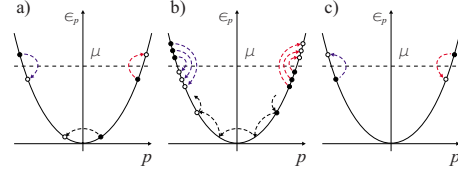


FIG. 2. (Color online) (a) Dominant three-particle collision which changes the number of right-moving electrons. (b) Equilibration mechanism, multistep backscattering of right mover into the left mover. At low temperatures each step δp in momentum space is on the order of $\sim T/v_F$. (c) Energy-conserving two-particle scattering process that violates conservation of momentum. This process is possible due to the presence of inhomogeneities.

scattered by electron excitations close to the Fermi level. These excitations are typically associated with a momentum change $\delta p \sim T/v_F \ll p_F$ due to Fermi blocking. Let us further characterize this process by introducing three-particle scattering rate $1/\tau_{eee}$, which can be approximated by a constant because the initial and final states both lie at the bottom of the band. Since the sign of δp varies in a random fashion from one collision to another the hole performs a Brownian motion in momentum space. The corresponding diffusion coefficient B can be readily estimated. The typical change of momentum of a hole over time t behaves as $(\Delta p)^2 \sim Bt$. As we assumed the hole changes its momentum by $\pm T/v_F$ once during the time τ_{eee} , so we conclude that $(\Delta p)^2 \sim (T/v_F)^2 t / \tau_{eee}$ for $t \gg \tau_{eee}$ and thus estimate

$$B \sim \frac{T^2}{v_F^2 \tau_{eee}}. \quad (33)$$

The change $\Delta \dot{N}^R$ in the number of right-moving electrons over the time $t \sim (\Delta p)^2/B$ for the segment of the wire Δx is given by the rate t^{-1} times the number of deep holes susceptible to be backscattered. The latter can be estimated from the probability to find a left- or right-moving hole $e^{-\mu^{L(R)}/T}$ and the number of states $\Delta p \Delta x / h$ available within the typical momentum range $\Delta p \sim \sqrt{mT}$ of the backscattering processes shown in Fig. 2(b). Taking into account that the scattering of left- and right-moving holes both contribute to \dot{N}^R , but with opposite signs, one finally estimates

$$\Delta \dot{N}^R \sim \frac{1}{t} \left(\frac{\Delta p \Delta x}{h} \right) (e^{-\mu^R/T} - e^{-\mu^L/T}) \approx - \frac{\Delta \mu \Delta x B}{h \sqrt{mT^3}} e^{-\mu/T} \quad (34)$$

with $\Delta \mu = \mu^R - \mu^L$. A careful calculation based on the kinetic equation gives³⁵

$$\frac{d\dot{N}^R}{dx} = - \frac{2\Delta\mu(x)}{h} \frac{e^{-\mu(x)/T}}{\ell_1(x)}, \quad \ell_1(x) = \frac{\sqrt{8\pi m T^3}}{B(x)}. \quad (35)$$

The expression for the diffusion coefficient B is model specific. Our preliminary estimate gives $B \propto T^3$ in the case of Coulomb interaction.

We continue now with the calculation of the rate \dot{Q}^R , which consists of two contributions,

$$\dot{Q}^R = \dot{Q}_b^R + \dot{Q}_p^R. \quad (36)$$

The first one \dot{Q}_b^R is related to the same backscattering events that change the number of right movers \dot{N}^R . Exploring the fact that both rates are caused by the same physical processes it was shown in Refs. 34 and 35 that there is a relation between \dot{N}^R and \dot{Q}_b^R , which we generalize here for the inhomogeneous case,

$$\frac{d\dot{Q}_b^R}{dx} = -2\mu(x) \frac{d\dot{N}^R}{dx}. \quad (37)$$

The logic behind this equation is as follows. The backscattering processes transform the unperturbed distribution of electrons into the partially equilibrated form (9) with nonvanishing $u(x)$. The two distributions differ most prominently at energies within $\sim T$ of the Fermi level. One can thus assume that all the right-moving electrons contributing to \dot{N}^R are removed from the vicinity of the right Fermi point and placed to the vicinity of the left one. Each such transfer reduces the momentum of the system by $2p_F$. The other electrons have to be scattered in the vicinities of the two Fermi points to accommodate this momentum change. In the special case of three-particle collisions, the transfer of electron from the right Fermi point to the left one is accomplished in a number of small steps with momentum change $\delta p \sim T/v_F$, and at each step one additional electron is scattered near each of the two Fermi points, see Fig. 2(b). As a result of the rearrangement of electrons near the two Fermi points, the local momentum change $2p_F$ of the backscattered electrons is distributed between the remaining right- and left-moving electrons, i.e., $\delta p^R + \delta p^L = 2p_F$. Thus the energy of the remaining right movers increases by $\delta E^R = v_F \delta p^R$ whereas that of the left movers decreases, $\delta E^L = -v_F \delta p^L$. Then, the conservation of energy requires $\delta p^R = \delta p^L = p_F$. In the end, the energy balance for the right-moving electrons consists of a loss of μ due to the removal of one particle from the Fermi level and a gain of $\delta E^R = v_F p_F = 2\mu$ due to the redistribution of momentum. As a result, for every right-moving electron that changes direction, $\Delta N^R = -1$, the right-movers energy increases by an amount $\Delta E^R = \mu$, so one concludes that $\dot{E}^R = -\mu \dot{N}^R$ or equivalently $\dot{Q}^R = -2\mu \dot{N}^R$. Equation (37) follows from here naturally if one applies the same argument but for the segment of wire Δx such that rates \dot{N}^R and \dot{Q}^R are accounted per unit of length in the inhomogeneous wire.

The other contribution \dot{Q}_p^R to the heat transferred by right movers in Eq. (36) is due to scattering processes that do not conserve momentum, see Fig. 2(c) for illustration. These two-body collisions are possible only in the inhomogeneous case. They do not change the number of right-moving electrons but do change their energy. It is expected that this rate is proportional to the velocity u of the electron liquid, $\dot{Q}_p^R \propto u$. Indeed, two-particle collisions of Fig. 2(c) involve a right mover with momentum $p \approx p_F$ and a left mover with momentum $p \approx -p_F$. For these electrons the drift term $pu \approx \pm p_F u$ in the distribution function (9) can be absorbed into the temperature \bar{T} , such that right movers can be considered

as being at an effective temperature $T^R \approx \bar{T}(1+u/v_F)$ while left movers at temperature $T^L \approx \bar{T}(1-u/v_F)$, to linear order in u .³³ According to the general principle of statistical mechanics thermalization between these subsystems involves the energy flow from “warmer” right movers to “colder” left movers that is proportional to the difference in temperatures between the two, $\dot{Q}_p^R \propto T^R - T^L \propto u$. An explicit microscopic calculation of the rate \dot{Q}_p^R done in Appendix A gives

$$\frac{d\dot{Q}_p^R}{dx} = -2\mu(x) \frac{n(x)u(x)}{\ell_{\text{in}}(x)}. \quad (38)$$

Here ℓ_{in} is a scattering length scale associated with these momentum-nonconserving collisions, Fig. 2(c),

$$\ell_{\text{in}}^{-1} = \frac{Y(x)}{16n(x)} \frac{T}{\mu(x)}, \quad (39)$$

where the parameter

$$Y(x) = \left\{ \left\{ \partial_x \left[\frac{\mathcal{V}_0 - \mathcal{V}_{2k_F(x)}}{\pi \hbar v_F(x)} \right] \right\}^2 + \left\{ \partial_x \left[\frac{\mathcal{V}_0}{\pi \hbar v_F(x)} \right] \right\}^2 + \left\{ \partial_x \left[\frac{\mathcal{V}_{2k_F(x)}}{\pi \hbar v_F(x)} \right] \right\}^2 \right\} \quad (40)$$

is expressed through the zero momentum and $2k_F$ Fourier components of the electronic interaction potential \mathcal{V} . The complete rate \dot{Q}^R is thus given by the sum of Eqs. (37) and (38) and we find

$$\frac{d\dot{Q}^R}{dx} = -2\mu(x) \frac{d\dot{N}^R}{dx} - 2\mu(x) \frac{n(x)u(x)}{\ell_{\text{in}}(x)}. \quad (41)$$

One should make two important comments regarding this result. First, the effect of inhomogeneity on resistivity of a quantum wire was recently addressed in Refs. 32 and 33 assuming that electrons are fully equilibrated and thus described by the distribution function (5). This assumption requires that three-body interaction processes which change the number of right-moving electrons dominate over the momentum-nonconserving scattering. However, in a situation where both interaction processes happen on a comparable time scale the system is frustrated with a finite value of $\Delta\mu \neq 0$ and thus $u \neq v_d = I/en$ so that electrons are described by the distribution, Eq. (9), which we used in our calculations.

To elucidate further the origin of the frustration it is important to emphasize that momentum-nonconserving scattering resists full equilibration of electrons into a single distribution, Eq. (5). These scattering processes reduce velocity u and thus prevent complete relaxation of the difference in chemical potentials $\Delta\mu$. Indeed, since the current I is fixed by the external circuit then according to Eq. (31) decrease in u implies increase in $\Delta\mu$. This effect is opposite to that of the equilibration processes due to three-particle collisions which tend to relax $\Delta\mu$.

Second, the correction to resistance of a nonuniform wire was obtained in studies^{32,33} by calculating the rate of momentum change \dot{P}^R for right-moving electrons. Note, how-

ever, that in an inhomogeneous system without translational invariance momentum is not a good quantum number. Indeed, the momentum change of right movers due to collisions depends on position and thus, is not the same for electrons inside the wire, where it was calculated,^{32,33} than that (actual change) in the leads. Our present scheme is free from this difficulty. As we show in the next section it is really the rate \dot{Q}^R , computed from the well-defined energy exchange, that is needed to determine the wire resistance and other transport coefficients.

IV. TRANSPORT COEFFICIENTS

We are set now for the calculation of transport coefficients in an inhomogeneous wire. Indeed, our basic equations (22) and (30) provide electric I and heat j_Q currents as the response to the applied bias V and temperature difference ΔT . Interaction effects are captured by the rates \dot{N}^R and \dot{Q}^R induced by particle collisions that encode change in the number of right-moving electrons and energy exchange between right and left movers, respectively. These rates are defined by Eqs. (35) and (41) which still contain the unknown difference between the chemical potentials of partially equilibrated right and left movers $\Delta\mu(x)$ and flow velocity $u(x)$. Equations (31) and (32) are the final ingredients that allow to establish the correspondence $\{V, \Delta T\} \rightleftharpoons \{I, j_Q\}$ and thus find transport coefficients of interest.

Technically one proceeds as follows. First, employing conservation of currents (recall that I and j_Q are constant along the wire) one can express $\Delta\mu(x)$ and $u(x)$ in terms of I and j_Q from Eqs. (31) and (32). Second, one brings these relations into Eqs. (35) and (41) to find the rates \dot{N}^R and \dot{Q}^R in terms of I and j_Q , which is possible to do in quadratures. Finally, Eqs. (22) and (30) define the desired correspondence $\{V, \Delta T\} \rightleftharpoons \{I, j_Q\}$. Employing this procedure we find two independent linear equations,

$$\frac{2e^2}{h}V = I(1 + r_1) - \frac{6}{\pi^2}ej_Qr_1\frac{\bar{\mu}}{T^2}, \quad (42)$$

$$\frac{2\pi^2e}{3h}T\Delta T = ej_Q\left[1 + \frac{12}{\pi^2}\frac{r_1\bar{\mu}^2 + \gamma^2}{T^2}\right] - 2Ir_1\bar{\mu}, \quad (43)$$

where we introduced the dimensionless parameter,

$$r_1 = \int_0^L \frac{dx}{\ell_1(x)} e^{-\mu(x)/T}, \quad (44)$$

which quantifies the rate of three-particle processes that change number of right-moving electrons [see Eq. (35)], as well as the weighted chemical potentials along the wire,

$$\bar{\mu} = \frac{1}{r_1} \int_0^L \frac{dx}{\ell_1(x)} \mu(x) e^{-\mu(x)/T}, \quad (45)$$

$$\bar{\mu}^2 = \frac{1}{r_1} \int_0^L \frac{dx}{\ell_1(x)} \mu^2(x) e^{-\mu(x)/T}, \quad (46)$$

$$\gamma^2 = \int_0^L \frac{dx}{\ell_{in}(x)} \mu^2(x). \quad (47)$$

From Eqs. (42) and (43) we find the resistance

$$R = \frac{V}{I} \Big|_{\Delta T=0} = \frac{h}{2e^2} [1 + r], \quad (48)$$

where

$$r = r_1 - \frac{r_1^2 \bar{\mu}^2}{\frac{\pi^2}{12} T^2 + r_1 \bar{\mu}^2 + \gamma^2} \quad (49)$$

and Peltier coefficient

$$\Pi = \frac{j_Q}{I} \Big|_{\Delta T=0} = \frac{\pi^2 T^2}{6e} \frac{r_1 \bar{\mu}}{\frac{\pi^2}{12} T^2 + r_1 \bar{\mu}^2 + \gamma^2}. \quad (50)$$

In addition we find the thermal conductance

$$K = \frac{j_Q}{\Delta T} \Big|_{I=0} = \frac{\pi^4 T^3}{18h} \frac{1}{\frac{\pi^2}{12} T^2 + r_1 \bar{\mu}^2 + \gamma^2} \quad (51)$$

and thermopower

$$S = - \frac{V}{\Delta T} \Big|_{I=0} = \frac{\pi^2 T}{6e} \frac{r_1 \bar{\mu}}{\frac{\pi^2}{12} T^2 + r_1 \bar{\mu}^2 + \gamma^2}. \quad (52)$$

Predictably, the Peltier coefficient, Eq. (50), and thermopower, Eq. (52), satisfy the Onsager relation $\Pi = ST$. Equations (48)–(52) are the main results of this paper. In the following we analyze these general expressions for a few modeling examples of inhomogeneities.

A. Uniform wire

First of all, we perform a consistency check for the case of a uniform wire, recently studied in Ref. 35. In the homogeneous case $U(x) \rightarrow 0$ all quantities defining R , Π , K , and S become coordinate independent: $\ell_1(x) \rightarrow \ell_1$ so that $\bar{\mu}^2 \equiv \bar{\mu}^2 = \mu^2$ and $r_1 \rightarrow (L/\ell_1)e^{-\mu/T}$. At the same time $\gamma \rightarrow 0$ which is a consequence of momentum conservation: in a uniform wire two-electron scattering processes, shown in Fig. 2(c), are not allowed. As a result, interaction-induced correction to the wire resistance, Eq. (49), reduces to

$$r = \frac{r_0 r_1}{r_0 + r_1}, \quad (53)$$

where $r_0 = \pi^2 T^2 / 12 \mu^2$. In order to establish a connection with the notations of Ref. 35 we invert resistance, Eq. (48), with r taken from Eq. (49), to get the conductance $G = R^{-1}$ to leading order in $T/\mu \ll 1$ and find

$$G = \frac{2e^2}{h} \left[1 - \frac{\pi^2 T^2}{12 \mu^2} \frac{L}{L + \ell_{eq}} \right], \quad (54)$$

where following Ref. 35 we have introduced the equilibration length,

$$\ell_{\text{eq}} = \frac{\pi^2 T^2}{12 \mu^2} \ell_1 e^{\mu/T}. \quad (55)$$

This result shows that for a long wire $L \gg \ell_{\text{eq}}$ the conductance saturates to its length independent value, which still exhibits noticeable power-law correction in temperature, $\delta G = -(2e^2/h)(\pi^2 T^2/12\mu^2)$, already mentioned in Sec. II [Eq. (6)]. This saturation of conductance is expected since the electronic system reaches full equilibrium. For short wires, $\ell_1 \ll L \ll \ell_{\text{eq}}$, the interaction-induced correction to conductance is exponentially small $\delta G = -(2e^2/h)(L/\ell_1)e^{-\mu/T}$ and scales linearly with the length of the wire. It is worth noting that Eq. (54) is only applicable to wires longer than ℓ_1 . This constraint comes from the approximations made when deriving the rate of change for the right-moving electrons \dot{N}^R in Eq. (35) (see Ref. 35 for details).

The Peltier coefficient and thermopower of a uniform wire follow from Eqs. (50) and (52), and read

$$\Pi = ST = \frac{\pi^2 T^2}{6e} \frac{L}{\mu L + \ell_{\text{eq}}}. \quad (56)$$

It shows that Π grows from exponentially small values at $L \ll \ell_{\text{eq}}$ to Π_{eq} quoted in Eq. (8) at $L \gg \ell_{\text{eq}}$. The thermal conductance behaves very differently though. One finds from Eq. (51) in the uniform limit,

$$K = \frac{2\pi^2 T}{3h} \frac{\ell_{\text{eq}}}{L + \ell_{\text{eq}}}. \quad (57)$$

At $L \ll \ell_{\text{eq}}$ one recovers the result, Eq. (2), for noninteracting wires, but as the length of the wire grows, K is suppressed as $1/L$ and vanishes for fully equilibrated wires, see our earlier discussion presented below Eq. (7). Equations (56) and (57) recover the corresponding results of Ref. 35.

B. Two wires in series

As the simplest prototype of nonuniform system we study two uniform wires of lengths L_1 and L_2 , with different densities, connected in series with each other. We ignore here small contribution to transport coefficients coming from γ , which is nonzero only in the near vicinity of the junction between the wires and vanishes everywhere else. Applying then Eqs. (45)–(47) to this setup we find for the interaction-induced resistance, Eq. (49),

$$r = \frac{\pi^2 T^2}{12} \frac{\frac{L_1}{\mu_1^2 \ell_{\text{eq}}^{(1)}} + \frac{L_2}{\mu_2^2 \ell_{\text{eq}}^{(2)}} + \left[\frac{1}{\mu_1} - \frac{1}{\mu_2} \right]^2 \frac{L_1 L_2}{\ell_{\text{eq}}^{(1)} \ell_{\text{eq}}^{(2)}}}{1 + \frac{L_1}{\ell_{\text{eq}}^{(1)}} + \frac{L_2}{\ell_{\text{eq}}^{(2)}}} \quad (58)$$

with chemical potentials $\mu_i = \mu - U_i$ and equilibration lengths $\ell_{\text{eq}}^{(i)} = (\pi^2 T^2/12\mu_i^2) \ell_1^{(i)} e^{\mu_i/T}$ within each wire $i=1,2$. It is of special interest to consider the limit when one wire is infinitely long. Upon taking the limit $L_1 \rightarrow \infty$ the expression for r reduces to

$$r \rightarrow \frac{\pi^2 T^2}{12 \mu_1^2} + \frac{\pi^2}{12} \left[\frac{T}{\mu_1} - \frac{T}{\mu_2} \right]^2 \frac{L_2}{\ell_{\text{eq}}^{(2)}}. \quad (59)$$

The first term of this formula corresponds to the residual resistance of the first wire. It is independent of its length L_1 , which is natural since in the limit $L_1 \rightarrow \infty$ electrons reach full equilibration and the interaction-induced resistance should saturate.³⁴ The other contribution to the resistance in Eq. (59) is due to the second wire, which, however, ceases to saturate even when $L_2 \gg \ell_{\text{eq}}^{(2)}$. Thus it remains proportional to the wire length L_2 and could be much larger than the first term. The absence of equilibration in the second wire is rather counterintuitive. This result forced us to reexamine more carefully continuity equations for electric and heat currents. We have found that it is not possible to match both I and j_Q in the wires while simultaneously imposing vanishing chemical-potential difference $\Delta\mu$ between right- and left-moving electrons. Indeed, let us suppose that in the limit $L_1 \gg \ell_{\text{eq}}^{(1)}$ and $L_2 \gg \ell_{\text{eq}}^{(2)}$ both wires are fully equilibrated so that $\Delta\mu=0$ in each wire. The current conservation then becomes $I = en_1 v_{d1} = en_2 v_{d2}$, where $v_{d1,2}$ is the drift velocity within each wire. According to Eq. (32) the heat current in this case can be written as $j_{Q1,2} = I(\pi^2/6e)(T^2/\mu_{1,2})$ and clearly $j_{Q1} \neq j_{Q2}$ since $\mu_1 \neq \mu_2$. The resolution of this controversy is possible only if $\Delta\mu \neq 0$ at least within one wire even though its length exceeds the corresponding equilibration length. In Appendix B we rederived Eq. (58) relying on conservation laws only.

Other transport coefficients do not show dramatic changes compared to a single uniform wire and their behavior follows expectedly as a natural generalization of Eqs. (56) and (57). We find for the Peltier coefficient of two connected uniform wires,

$$\Pi = \frac{\pi^2 T^2}{6e} \frac{\frac{L_1}{\mu_1 \ell_{\text{eq}}^{(1)}} + \frac{L_2}{\mu_2 \ell_{\text{eq}}^{(2)}}}{1 + \frac{L_1}{\ell_{\text{eq}}^{(1)}} + \frac{L_2}{\ell_{\text{eq}}^{(2)}}}. \quad (60)$$

Π saturates to $\pi^2 T^2/6e\mu_{1(2)}$ depending on which wire is fully equilibrated. The thermal conductance is found to be

$$K = \frac{2\pi^2 T}{3h} \frac{1}{1 + \frac{L_1}{\ell_{\text{eq}}^{(1)}} + \frac{L_2}{\ell_{\text{eq}}^{(2)}}}, \quad (61)$$

which is natural generalization of Eq. (57).

C. Wire with long-range disorder

We now study more generic models of a nonuniform wire. We assume only that disorder variations happen on the large spatial scale, $k_F^{-1} \ll b \ll L$, and concentrate on the case $\gamma^2 \gg T^2$. In this case the $\pi^2 T^2/12$ term in Eq. (49) can be ignored, and the interaction-induced resistance of the wire r can be written as

$$r = r_1 \frac{r_1 \overline{\delta\mu^2} + \gamma^2}{r_1 \overline{\mu^2} + \gamma^2}, \quad (62)$$

where we introduced $\overline{\delta\mu^2} = \overline{\mu^2} - \overline{\mu}^2$. Expression (62) is applicable to any realization of long-range disorder potential. We

now apply it to two special cases which allow simple analytical solution.

1. Weak disorder

First, let us assume that amplitude U_0 of variations in the inhomogeneity potential along the wire is small, $U_0 \ll T$. It turns out that Eq. (62) covers three distinct regimes depending on the temperature. At lowest temperatures $T \ll T_1$, where

$$T_1 \approx \frac{\mu}{2 \ln \left[\frac{\mathcal{V}_0}{\hbar v_F} k_F b \frac{\mu}{U_0} \right]}, \quad (63)$$

three-particle equilibration processes are weak due to exponential suppression $e^{-\mu/T}$ of the scattering near bottom of the band.⁴⁰ In this regime $r_1 \delta\mu^2 \ll r_1 \bar{\mu}^2 \ll \gamma^2$ and the resistance of the wire, Eq. (62), is given by r_1 . It then follows from Eq. (44) that to leading order in $U_0 \ll T$,

$$r = \rho_1 L, \quad \rho_1 = \frac{1}{\ell_1} e^{-\mu/T}, \quad T \ll T_1, \quad (64)$$

where ρ_1 has the meaning of dimensionless resistivity of the wire. It is interesting to compare this result with Eq. (54) obtained for a uniform wire. In the limit $L \ll \ell_{\text{eq}}$ using Eqs. (48) and (55) we extract from Eq. (54) the correction to resistance $r = (L/\ell_1) e^{-\mu/T}$ which coincides with Eq. (64). However, there is an important difference in the applicability of this result to uniform and disordered wires. In the case of uniform wires $r = (L/\ell_1) e^{-\mu/T}$ applies only in the short wire limit $L \ll \ell_{\text{eq}}$ or equivalently at temperatures $T \ll T_1^* = \mu/\ln(L/\ell_1)$. For longer wires r saturates to the length-independent value $r = \pi^2 T^2 / 12 \mu^2$, and thus always remains smaller than contact resistance, $r \ll 1$. In contrast, in the case of disordered wires the result, Eq. (64), does not rely on the assumption that $L \ll \ell_{\text{eq}}$. The crossover temperature, Eq. (63), is controlled by disorder and does not depend on L . Thus, although the resistivity ρ_1 is small, for a sufficiently long wire the total resistance $r = \rho_1 L$ can be large, $r \gg 1$.

At higher temperatures $T \gg T_1$ (strong equilibration) the resistance is given by the sum of two terms,

$$r = r_1 \frac{\delta\mu^2}{\bar{\mu}^2} + \frac{\gamma^2}{\bar{\mu}^2}. \quad (65)$$

The contribution of the momentum-nonconserving two-body collisions induced by disorder, the second term in Eq. (65), dominates in the temperature regime $T_1 \ll T \ll T_2$, where

$$T_2 \approx \frac{\mu}{2 \ln \left[\frac{\mathcal{V}_0}{\hbar v_F} k_F b \right]}. \quad (66)$$

In this case the resistance of the wire is given by^{32,33}

$$r = \rho_2 L, \quad \rho_2 = \frac{\langle Y \rangle T}{16 n \mu}, \quad T_1 \ll T \ll T_2. \quad (67)$$

Here we used Eqs. (39) and (47), set $\bar{\mu}^2 = \bar{\mu}^2 = \mu^2$ to the leading order in $U_0 \ll T$, and $\langle \dots \rangle = \int_0^L \frac{dx}{L} (\dots)$ implies averaging along the wire. In Refs. 32 and 33 the same result for resis-

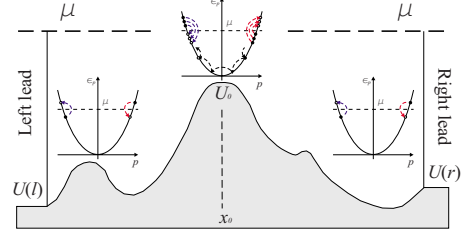


FIG. 3. (Color online) Enhanced equilibration due to three-particle collisions in the wire segment of length ℓ_T where the inhomogeneity potential is maximal. A larger value of $e^{-\mu(x)/T}$ near $x \sim x_0$ favors stronger electron backscattering in accordance with Eq. (35). In contrast, momentum-nonconserving two-particle collisions occur throughout the wire.

tivity was derived assuming that electrons are fully equilibrated. Here we find that in fact applicability conditions for ρ_2 are more strict and Eq. (67) dominates only in the temperature range $T_1 \ll T \ll T_2$. At higher temperatures the resistance is governed by the first term in Eq. (65),

$$r = \rho_3 L, \quad \rho_3 = \frac{\pi^4 T^3}{18 h \mu^4} \frac{\langle \delta\mu^2 \rangle}{\mu}, \quad T_2 \ll T \ll \mu, \quad (68)$$

where $\langle \delta\mu^2 \rangle = \langle U^2 \rangle$ since $\mu(x) = \mu - U(x)$. In Eq. (68) we introduced thermal conductivity of a wire $\kappa = (2\pi^2 T / 3h) \ell_{\text{eq}}$. Indeed, according to Eq. (57) for long wires, $L \gg \ell_{\text{eq}}$, we have $K = \kappa / L$.

Equation (68) shows that resistance r is determined by the magnitude of the disorder potential rather than its gradients. Similar to the case of two wires in series, considered in Sec. IV B, this feature can be traced back to the fact that in the inhomogeneous wire electrons never reach full equilibration no matter how long the wire is. It is also interesting to note that resistance of the wire in the regime of strong equilibration [Eq. (68)] can be understood from purely hydrodynamic considerations.⁴¹

2. Strong disorder

We now relax the assumption of small variations in the amplitude of $U(x)$. Consider the case when generally smooth profile of the inhomogeneity potential has a well-defined maximum $U_0 \gg T$ at some point x_0 inside the wire, see Fig. 3. We assume that such strong fluctuation of $U(x)$ happens in only one place along the wire. This situation is likely to occur in practice due to the presence of charged impurities in the substrate and/or uneven screening of the nearby gates.

Since $e^{-\mu(x)/T}$ is largest near x_0 and $dN^R/dx \propto e^{-\mu(x)/T}$, see Eq. (35), it is natural to expect that three-particle equilibration processes are significantly enhanced in the part of the wire where $U(x)$ reaches its maximum. Thus this region of the wire gives dominant contribution to resistance at lowest temperatures when $\gamma^2 \gg r_1 \bar{\mu}^2 \gg r_1 \delta\mu^2$. Assuming smoothness of $\ell_1(x)$ at $x \sim x_0$ as compared to the sharp $e^{-\mu(x)/T}$ we can compute the resistance $r = r_1$ by applying the saddle point approximation to Eq. (44),

$$r = \frac{\ell_T}{\ell_1} e^{-\mu_0/T}, \quad \ell_T = \sqrt{\frac{2\pi T}{|U''(x_0)|}}. \quad (69)$$

Here $\mu_0 = \mu - U(x_0)$, $\ell_1 = \ell_1(x_0)$, and ℓ_T is the thermal length associated with the curvature of the potential $U(x)$ near x_0 .

In contrast to the equilibration processes, dominated by scattering near x_0 , momentum-nonconserving two-particle collisions occur throughout the wire. Resistance of the wire is controlled by the latter at intermediate temperatures when $r_1 \bar{\mu}^2 \gg \gamma^2 \gg r_1 \delta \mu^2$. For this regime

$$r = \frac{TL}{16\mu_0^2} \left\langle \frac{Y(x)\mu(x)}{n(x)} \right\rangle, \quad (70)$$

which is analogous to Eq. (67). The only difference is that due to strong variations in $U(x)$ spatial averaging in Eq. (70) involves not only $Y(x)$.

At higher temperatures, when $r_1 \bar{\mu}^2 \gg r_1 \delta \mu^2 \gg \gamma^2$, resistance is again dominated by the scattering processes near the top of the inhomogeneity potential. In this regime the first term in Eq. (65), determined by the amplitude fluctuations of $U(x)$ rather than its gradient, gives the leading contribution,

$$r = \frac{\pi^2 T^4}{24 \mu_0^4 \ell_{\text{eq}}(x_0)} \ell_T. \quad (71)$$

Here we used $\delta \mu^2 / \bar{\mu}^2 = T^2 / 2\mu_0^2$ found within saddle-point approximation from Eqs. (45) and (46) to leading order in $T/\mu_0 \ll 1$. Equation (71) is analogous to Eq. (68) with the thermal length ℓ_T effectively playing the role of the system size.

V. SUMMARY

In this paper we studied the transport properties of weakly interacting one-dimensional electrons in the presence of inhomogeneities. In this system equilibration is strongly restricted by the phase space available for electron scattering and conservation laws. The resulting equilibration length $\ell_{\text{eq}} \propto e^{\mu/T}$ is exponentially large at low temperatures and the partially equilibrated state is more likely to be realized than the fully equilibrated one. Furthermore, inhomogeneities present in the wire themselves resist equilibration of electrons due to momentum-nonconserving two-particle collisions.

Our main results are expressions (48)–(52) for the resistance, Peltier coefficient, thermal conductance and thermopower. We find that the combined effect of interactions and inhomogeneities can dramatically increase the resistance of the wire. For the long enough wire the induced correction could be much greater than the contact resistance of noninteracting electrons $h/2e^2$. This is in contrast to the uniform case where interaction-induced correction to resistance saturates for $L \gg \ell_{\text{eq}}$ and remains small as $(T/\mu)^2$ compared to the resistance $h/2e^2$ of a noninteracting wire.

The combined effect of interactions and inhomogeneities is different for thermoelectric coefficients. On the one hand, when temperature increases, Peltier coefficient and thermopower grow from exponentially small values, Eq. (3), to $\Pi = TS \approx (\pi^2 T^2 / 6e)(\bar{\mu} / \mu^2)$. On the other, this enhancement

is not as dramatic as in the case of resistance. Indeed, the difference between the saturated values of Peltier coefficient and thermopower in inhomogeneous wire as compared to that in the uniform wires is only in the appearance of the renormalized factor $\bar{\mu} / \mu^2$ instead of the inverse chemical potential $1/\mu$, see Eq. (8). Conversely, the thermal conductance of the wire decreases due to the equilibration from its noninteracting value, Eq. (2), to zero at $L \gg \ell_{\text{eq}}$.

The lack of complete electronic equilibration in the inhomogeneous quantum wires, which is another central observation of our work, warrants additional discussion. The notion of equilibration appears naturally since initially right- and left-moving electrons entering the wire from the left and right lead, respectively, are at different equilibria with respect to each other due to the applied bias or temperature difference. In the case of weak interactions three-particle collisions constitute the leading-order relaxation process. Although the corresponding relaxation rate is slow (or equivalently relaxation length is large) due to the required scattering through the bottom of the band, complete equilibration between the right and left movers is nevertheless possible in a *homogeneous* wire. Once the length of the wire becomes large such that exponential suppression of the equilibration effects is compensated by a large system size, $L \gg \ell_{\text{eq}} \sim e^{\mu/T}$, the relaxation of the electron system becomes significant. Right and left movers eventually equilibrate to the single distribution, Eq. (5). When viewed in a reference frame moving together with electrons this distribution is simply the equilibrium Fermi function. Thus, in the stationary frame this is distribution with a boost.

In the presence of spatial *inhomogeneities* full equilibration is impeded. This is most transparent when momentum-nonconserving two-body collisions are present, which unlike three-particle processes favor electron distribution function without boost, Eq. (4). As a result, the electron system is frustrated due to competition between two scattering processes and an intermediate distribution, Eq. (9), is established.

Interestingly, lack of full equilibration is a more general characteristic of inhomogeneous quantum wires, which is a consequence of conservation laws. More precisely, in inhomogeneous wires it is generally not possible to reconcile full equilibration with conservation of energy. Technically speaking, this observation comes from the fact that in the fully equilibrated state all currents are proportional to the drift velocity. In the linear-response regime one therefore has only a single parameter v_d to simultaneously satisfy uniformity of particle and heat currents along the wire, imposed by conservation laws. In inhomogeneous wires this is generally not possible since for $\Delta\mu=0$ the ratio $j_Q/I = \pi^2 T^2 / 6e\mu(x)$ is not constant along the wire. Thus the electron liquid must remain in the state of partial equilibration with $\Delta\mu \neq 0$. We have illustrated this point explicitly by considering the simplest example of inhomogeneity, namely, a junction of two uniform wires with mismatched densities. In the general case of a wire with long-range disorder the consequence of the partially equilibrated state is that interaction-induced correction to resistance of the wire is determined by the amplitude of the variations in inhomogeneity potential, rather than its gradients. This correction may be large compared to the resistance $h/2e^2$ of noninteracting wires.

ACKNOWLEDGMENTS

We are grateful to B. L. Altshuler, A. V. Andreev, A. P. Dmitriev, Y. M. Galperin, I. V. Gornyi, D. G. Polyakov, and B. Shklovskii for helpful discussions. This work at ANL was supported by the U.S. Department of Energy, Office of Science, under Contract No. DE-AC02-06CH11357.

APPENDIX A: DETAILS ON THE CALCULATION OF \dot{Q}_p^R

For the inhomogeneous wire we describe the electron-electron interaction responsible for two-body scattering by its general translationally noninvariant form,

$$\mathcal{V}(x, x') \Rightarrow \mathcal{V}\left(x - x', \frac{x + x'}{2}\right). \quad (\text{A1})$$

As a function of its first argument \mathcal{V} is assumed to be Coulombic in nature and thus short ranged with variations on the scale of a certain screening length due to the nearby gates. The inhomogeneity is captured by the second argument with a corresponding variations in \mathcal{V} on the length scale b , large compared to both, the Fermi wavelength λ_F and the range of screening. Our starting point for the energy-transfer rate from the right movers for the segment of the wire $\hbar v_F/T \ll \Delta x \ll b$ is the following golden-rule expression,

$$\begin{aligned} \Delta \dot{Q}_p^R(x) = & -\frac{2\pi}{\hbar} \int \frac{d\epsilon_p d\epsilon_{p'} d\epsilon_k d\epsilon_{k'}}{(2\pi)^4} |\mathcal{V}(\epsilon_p, \epsilon_k; \epsilon_{p'}, \epsilon_{k'})|^2 \\ & \times \delta(\epsilon_p + \epsilon_k - \epsilon_{p'} - \epsilon_{k'}) (\epsilon_p - \epsilon_{p'} + \epsilon_{k'} - \epsilon_k) \\ & \times \{f^R(\epsilon_p)[1 - f^R(\epsilon_{p'})]f^L(\epsilon_k)[1 - f^L(\epsilon_{k'})] \\ & - f^R(\epsilon_{p'})[1 - f^R(\epsilon_p)]f^L(\epsilon_{k'})[1 - f^L(\epsilon_k)]\}, \quad (\text{A2}) \end{aligned}$$

where the matrix element

$$|\mathcal{V}|^2 = |\mathcal{V}_{\parallel}|^2 + |\mathcal{V}_{1\perp}|^2 + |\mathcal{V}_{2\perp}|^2, \quad (\text{A3})$$

includes three possible scattering processes of spinfull electrons from the initial state (ϵ_p, ϵ_k) into the final state $(\epsilon_{p'}, \epsilon_{k'})$. These matrix elements will be calculated on the basis of semiclassical wave functions,

$$\psi_{\epsilon, \pm}(x) = \frac{1}{\sqrt{\hbar v_F(x)}} \exp\left[\pm \frac{i}{\hbar} \int_0^x dx' \sqrt{2m[\epsilon - U(x)]}\right], \quad (\text{A4})$$

which are eigenstates of the free Hamiltonian $[-\hbar^2 \partial_x^2 / 2m + U(x)]\psi_{\epsilon}(x) = \epsilon \psi_{\epsilon}(x)$ normalized according to $\int dx \psi_{\epsilon, \pm}(x) \psi_{\epsilon', \pm}^*(x) = 2\pi \delta(\epsilon - \epsilon')$ and $v_{\epsilon}(x) = p_{\epsilon}(x)/m$. The subscripts \pm refer to the right/left branches and we also ignored the backscattering wave since it only leads to an exponentially small contribution for $k_F b \gg 1$. Focusing on the states close to the Fermi energy, we can simplify the expression for the eigenstates, Eq. (A4), of the free Hamiltonian into

$$\psi_{\epsilon, \pm}(x) \approx \psi_{\mu, \pm}(x) \exp\left[\pm i(\epsilon - \mu) \int_0^x \frac{dx'}{\hbar v_F(x')}\right], \quad (\text{A5})$$

where $\psi_{\mu, \pm}(x)$ is obtained from Eq. (A4) by setting $\epsilon = \mu$. This allows us to find, for example, the matrix element \mathcal{V}_{\parallel} to first order in the interaction,

$$\begin{aligned} \mathcal{V}_{\parallel}(\epsilon_p, \epsilon_k; \epsilon_{p'}, \epsilon_{k'}) & = \int_x^{x+\Delta x} dX \frac{\exp\left[i \int_0^X dx' \frac{\epsilon_{p'} - \epsilon_p + \epsilon_k - \epsilon_{k'}}{\hbar v_F(x')}\right]}{[\hbar v_F(X)]^2} \\ & \times \int_{-\Delta x}^{\Delta x} dy \mathcal{V}(y, X) [1 - e^{-2ik_F(X)y}]. \quad (\text{A6}) \end{aligned}$$

Here we introduced center of mass $X = (x + x')/2$ and relative $y = x - x'$ coordinates, and $k_F(X) = p_F(X)/\hbar$.

Using Eq. (9) and expanding the occupation factors $f^{R/L}(\epsilon)$ in Eq. (A2) to linear order in $u(x)$, and splitting the energy-conserving delta function into two as $\int d\omega \delta(\epsilon_p - \epsilon_{p'} - \omega) \delta(\epsilon_k - \epsilon_{k'} + \omega)$, we can complete $\epsilon_{p'}$ and $\epsilon_{k'}$ integrations and find

$$\begin{aligned} \Delta \dot{Q}_{p\parallel}^R(x) = & -\frac{2\pi T u(x)}{\hbar v_F(x)} \int \frac{d\omega d\epsilon_p d\epsilon_k}{(2\pi)^4} |\mathcal{V}_{\parallel}(\omega)|^2 \frac{4\omega^2}{T^2} f(\epsilon_p) \\ & \times [1 - f(\epsilon_p - \omega)] f(\epsilon_k) [1 - f(\epsilon_k + \omega)], \quad (\text{A7}) \end{aligned}$$

where $f(\epsilon)$ is now the equilibrium Fermi function. The corresponding matrix element in these notations reads

$$\begin{aligned} \mathcal{V}_{\parallel}(\omega) = & \int_x^{x+\Delta x} dX \frac{\mathcal{V}_0(X) - \mathcal{V}_{2k_F}(X)}{[\hbar v_F(X)]^2} \\ & \times \exp\left[-2i\omega \int_0^X \frac{dx'}{\hbar v_F(x')}\right]. \quad (\text{A8}) \end{aligned}$$

The shortened forms \mathcal{V}_0 and \mathcal{V}_{2k_F} correspond to the zero momentum and $2k_F$ Fourier components of the potential $\mathcal{V}(y, X)$ with respect to its first variable y defined as $\mathcal{V}_0(X) = \int dy \mathcal{V}(y, X)$ and $\mathcal{V}_{2k_F}(X) = \int dy \mathcal{V}(y, X) e^{-2ik_F(X)y}$. At this stage $\epsilon_{p'}$ and $\epsilon_{k'}$ integrations can be completed by noticing that

$$f(\epsilon)[1 - f(\epsilon \pm \omega)] = \frac{f(\epsilon) - f(\epsilon \pm \omega)}{1 - e^{\mp \omega/T}},$$

$$\int_{-\infty}^{+\infty} d\epsilon [f(\epsilon) - f(\epsilon \pm \omega)] = \pm \omega. \quad (\text{A9})$$

As a result, we obtain following expression for the energy-transfer rate:

$$\begin{aligned} \Delta \dot{Q}_{\text{pl}}^R(x) = & -\frac{Tu(x)}{8\pi\hbar v_F(x)} \int \int_x^{x+\Delta x} dX_1 dX_2 \left[\frac{\mathcal{V}_0(X_1) - \mathcal{V}_{2k_F}(X_1)}{\pi\hbar v_F(X_1)} \right] \\ & \times \left[\frac{\mathcal{V}_0(X_2) - \mathcal{V}_{2k_F}(X_2)}{\pi\hbar v_F(X_2)} \right] \int_{-\infty}^{+\infty} d\omega \left(\frac{\omega^2/T}{\sinh \frac{\omega}{2T}} \right)^2 \\ & \times \frac{\exp \left[-2i\omega \int_{X_1}^{X_2} \frac{dx'}{\hbar v_F(x')} \right]}{\hbar v_F(X_1)\hbar v_F(X_2)}. \end{aligned} \quad (\text{A10})$$

Being interested in the temperature range $T \gg \hbar v_F/b$, where the exponential is rapidly oscillating, we write

$$\begin{aligned} & \frac{4\omega^2}{\hbar v_F(X_1)\hbar v_F(X_2)} \exp \left[-2i\omega \int_{X_1}^{X_2} \frac{dx'}{\hbar v_F(x')} \right] \\ & = \frac{\partial^2}{\partial X_1 \partial X_2} \exp \left[-2i\omega \int_{X_1}^{X_2} \frac{dx'}{\hbar v_F(x')} \right], \end{aligned} \quad (\text{A11})$$

integrate by parts over X_1 and X_2 and complete the remaining energy integral, which gives $4\pi\hbar v_F(x)\delta(X_1-X_2)$. Due to the delta function, one spatial integration is thus removed and we find as the final result, Eq. (38)–(40), where all three scattering channels were included.

APPENDIX B: SERIES RESISTANCE OF TWO UNIFORM WIRES FROM THE CONSERVATION LAWS

The resistance of a junction between two uniform wires with different densities can be found simply by combining conservation laws for the currents with the microscopic equations for \dot{N}^R and \dot{Q}^R . Our starting point is Eq. (22) which is naturally generalized to the case of two wires connected in series,

$$\frac{2e^2V}{h} = I - e\dot{N}_1^R - e\dot{N}_2^R, \quad (\text{B1})$$

where $\dot{N}_{1,2}^R$ correspond to the change in the number of right movers within each segment of the wire. Similarly, generalization of Eq. (30) for the heat balance reads

$$\frac{2\pi^2 T \Delta T}{3h} = j_Q - \dot{Q}_1^R - \dot{Q}_2^R. \quad (\text{B2})$$

Equation (B1) defines the resistance, Eq. (48), where interaction-induced correction is given by

$$r = -\frac{e\dot{N}_1^R}{I} - \frac{e\dot{N}_2^R}{I}. \quad (\text{B3})$$

We use now Eq. (35) in the limit of uniform wires to express rates $\dot{N}_{1,2}^R$ as

$$e\dot{N}_i^R = -I(1 - \alpha_i) \frac{L_i}{\lambda_i}, \quad \lambda_i = \ell_1^{(i)} e^{\mu_i/T}, \quad i = 1, 2, \quad (\text{B4})$$

by introducing the new quantity $\alpha_i = u_i/v_d$, which has meaning of the degree of equilibration. Indeed, $\alpha=0$ corresponds to the limit of no equilibration, such that the current is determined solely by $\Delta\mu$, while $\alpha=1$ corresponds to full equilibration where $I = env_d$. This notation makes it possible to express $\Delta\mu_i$ in Eq. (31) in terms of α_i as $I = 2e\Delta\mu_i/h + \alpha_i I$, which was used in Eq. (B4).

Conservation of the heat current, Eq. (32),

$$ej_Q = \frac{\pi^2 T^2}{6 \mu_1} \alpha_1 I = \frac{\pi^2 T^2}{6 \mu_2} \alpha_2 I \quad (\text{B5})$$

imposes a constraint $\alpha_1/\mu_1 = \alpha_2/\mu_2 \equiv \chi$, which must be constant. With the help of Eq. (B4) the expression for r in Eq. (B3) can be rewritten as

$$r = \frac{L_1}{\lambda_1} + \frac{L_2}{\lambda_2} - \chi \left(\mu_1 \frac{L_1}{\lambda_1} + \mu_2 \frac{L_2}{\lambda_2} \right). \quad (\text{B6})$$

where the unknown quantity χ is yet to be determined. The way to find it is from the energy conservation. Recall, that \dot{Q}^R and \dot{N}^R are related to each other by Eq. (37) since they are caused by the same scattering mechanism. For the uniform wire Eq. (37) reads $\dot{Q}_i^R = -2\mu_i \dot{N}_i^R$. As a result, for Eq. (B2) in the case of no temperature bias ($\Delta T=0$) we have

$$j_Q + 2\mu_1 \dot{N}_1^R + 2\mu_2 \dot{N}_2^R = 0. \quad (\text{B7})$$

Inserting here Eqs. (B4) and (B5) one finds

$$\frac{\pi^2 T^2}{6} \chi - 2\mu_1(1 - \chi\mu_1) \frac{L_1}{\lambda_1} - 2\mu_2(1 - \chi\mu_2) \frac{L_2}{\lambda_2} = 0, \quad (\text{B8})$$

which allows to find χ explicitly,

$$\chi = \frac{2\mu_1 \frac{L_1}{\lambda_1} + 2\mu_2 \frac{L_2}{\lambda_2}}{\frac{\pi^2 T^2}{6} + 2\mu_1^2 \frac{L_1}{\lambda_1} + 2\mu_2^2 \frac{L_2}{\lambda_2}}. \quad (\text{B9})$$

Finally, using this χ in Eq. (B6) and reexpressing λ through the equilibration length as $\lambda_i = (12\mu_i^2/\pi^2 T^2)\ell_{\text{eq}}^{(i)}$ one recovers Eq. (58). We thus conclude that Eq. (58) is just a consequence of the conservation laws.

¹B. J. van Wees, H. van Houten, C. W. J. Beenakker, J. G. Williamson, L. P. Kouwenhoven, D. van der Marel, and C. T. Foxon, *Phys. Rev. Lett.* **60**, 848 (1988).

²D. A. Wharam, T. J. Thornton, R. Newbury, M. Pepper, H.

Ahmed, J. E. F. Frost, D. G. Hasko, D. C. Peacock, D. A. Ritchie, and G. A. C. Jones, *J. Phys. C* **21**, L209 (1988).

³R. Landauer, *IBM J. Res. Dev.* **1**, 223 (1957); *Philos. Mag.* **21**, 863 (1970).

- ⁴D. L. Maslov and M. Stone, *Phys. Rev. B* **52**, R5539 (1995).
- ⁵V. V. Ponomarenko, *Phys. Rev. B* **52**, R8666 (1995).
- ⁶I. Safi and H. J. Schulz, *Phys. Rev. B* **52**, R17040 (1995).
- ⁷K. J. Thomas, J. T. Nicholls, M. Y. Simmons, M. Pepper, D. R. Mace, and D. A. Ritchie, *Phys. Rev. Lett.* **77**, 135 (1996).
- ⁸K. J. Thomas, J. T. Nicholls, N. J. Appleyard, M. Y. Simmons, M. Pepper, D. R. Mace, W. R. Tribe, and D. A. Ritchie, *Phys. Rev. B* **58**, 4846 (1998).
- ⁹K. J. Thomas, J. T. Nicholls, M. Pepper, W. R. Tribe, M. Y. Simmons, and D. A. Ritchie, *Phys. Rev. B* **61**, R13365 (2000).
- ¹⁰A. Kristensen, H. Bruus, A. E. Hansen, J. B. Jensen, P. E. Lindelof, C. J. Marckmann, J. Nygard, C. B. Sorensen, F. Beuscher, A. Forchel, and M. Michel, *Phys. Rev. B* **62**, 10950 (2000).
- ¹¹D. J. Reilly, G. R. Facer, A. S. Dzurak, B. E. Kane, R. G. Clark, P. J. Stiles, R. G. Clark, A. R. Hamilton, J. L. O'Brien, N. E. Lumpkin, L. N. Pfeiffer, and K. W. West, *Phys. Rev. B* **63**, 121311(R) (2001).
- ¹²S. M. Cronenwett, H. J. Lynch, D. Goldhaber-Gordon, L. P. Kouwenhoven, C. M. Marcus, K. Hirose, N. S. Wingreen, and V. Umansky, *Phys. Rev. Lett.* **88**, 226805 (2002).
- ¹³R. de Picciotto, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, *Phys. Rev. B* **72**, 033319 (2005).
- ¹⁴L. P. Rokhinson, L. N. Pfeiffer, and K. W. West, *Phys. Rev. Lett.* **96**, 156602 (2006).
- ¹⁵R. Crook, J. Prance, K. J. Thomas, S. J. Chorley, I. Farrer, D. A. Ritchie, M. Pepper, and C. G. Smith, *Science* **312**, 1359 (2006).
- ¹⁶C. K. Wang and K. F. Berggren, *Phys. Rev. B* **54**, R14257 (1996).
- ¹⁷H. Bruus and K. Flensberg, *Semicond. Sci. Technol.* **13**, A30 (1998).
- ¹⁸B. Spivak and F. Zhou, *Phys. Rev. B* **61**, 16730 (2000).
- ¹⁹Y. Meir, K. Hirose, and N. S. Wingreen, *Phys. Rev. Lett.* **89**, 196802 (2002).
- ²⁰Y. Tokura and A. Khaetskii, *Physica E* **12**, 711 (2002).
- ²¹G. Seelig and K. A. Matveev, *Phys. Rev. Lett.* **90**, 176804 (2003).
- ²²T. Rejec and Y. Meir, *Nature (London)* **442**, 900 (2006).
- ²³H. Bruus, V. V. Cheianov, and K. Flensberg, *Physica E* **10**, 97 (2001).
- ²⁴K. A. Matveev, *Phys. Rev. B* **70**, 245319 (2004).
- ²⁵D. Meidan and Y. Oreg, *Phys. Rev. B* **72**, 121312(R) (2005).
- ²⁶O. F. Syljuåsen, *Phys. Rev. Lett.* **98**, 166401 (2007).
- ²⁷D. A. Bagrets, I. V. Gornyi, A. D. Mirlin, and D. G. Polyakov, *Semiconductors* **42**, 994 (2008).
- ²⁸A. M. Lunde, A. De Martino, A. Schulz, R. Egger, and K. Flensberg, *New J. Phys.* **11**, 023031 (2009).
- ²⁹Y. M. Sirenko, V. Mitin, and P. Vasilopoulos, *Phys. Rev. B* **50**, 4631 (1994).
- ³⁰M. Khodas, M. Pustilnik, A. Kamenev, and L. I. Glazman, *Phys. Rev. B* **76**, 155402 (2007).
- ³¹A. M. Lunde, K. Flensberg, and L. I. Glazman, *Phys. Rev. B* **75**, 245418 (2007).
- ³²J. Rech and K. A. Matveev, *Phys. Rev. Lett.* **100**, 066407 (2008).
- ³³J. Rech and K. A. Matveev, *J. Phys.: Condens. Matter* **20**, 164211 (2008).
- ³⁴J. Rech, T. Micklitz, and K. A. Matveev, *Phys. Rev. Lett.* **102**, 116402 (2009).
- ³⁵T. Micklitz, J. Rech, and K. A. Matveev, *Phys. Rev. B* **81**, 115313 (2010).
- ³⁶D. B. Gutman, Y. Gefen, and A. D. Mirlin, *Phys. Rev. B* **80**, 045106 (2009); **81**, 085436 (2010).
- ³⁷A. M. Lunde, S. E. Nigg, and M. Büttiker, *Phys. Rev. B* **81**, 041311(R) (2010).
- ³⁸C. Altimiras, H. le Sueur, U. Gennser, A. Cavanna, D. Mailly, and F. Pierre, *Nat. Phys.* **6**, 34 (2010).
- ³⁹P. Degiovanni, Ch. Grenier, G. Fève, C. Altimiras, H. le Sueur, and F. Pierre, *Phys. Rev. B* **81**, 121302(R) (2010).
- ⁴⁰In the limit, where the equilibration due to triple collisions is weaker than that due to pair collisions in the presence of forward-scattering disorder, the resistivity was also studied by D. N. Aristov, A. P. Dmitriev, I. V. Gornyi, and D. G. Polyakov (unpublished).
- ⁴¹A. V. Andreev and B. Z. Spivak (private communication).